The uncertainty principle enables non-classical dynamics in an interferometer

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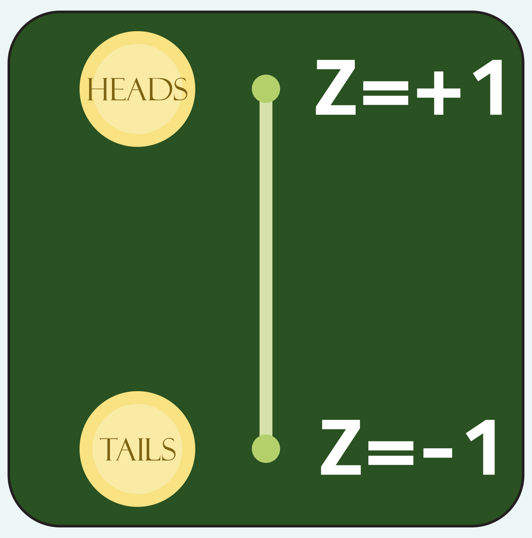
1. Introduction to the framework

It is possible to take an operational approach to describe the **state** of a physical system by listing the complete set of probabilities of the different outcomes of the next measurement made on the system. In a classical system, the state is uniquely specified by the probabilities associated with a single measurement, but in general we require more than one measurement.

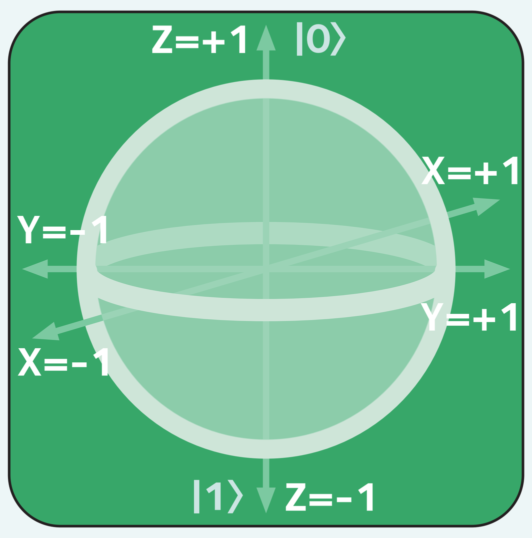
We can write these statistics as a set of expectation values for binary measurements. We label the two outputs +1 and -1 for each measurement. The entire state may be written as a vector such as (<X>, <Y>, <Z>), which we can represent as a geometric point in a Euclidean space. The set of all possible states allowed by a theory combine to form a **state-space**.



Classical

The two end points of this line correspond to when the coin is known to definitely be in a *heads* state (or a *tails* state).

The connecting line between shows various states of uncertainty, with the midpoint (<Z> = 0) corresponding to total lack of knowledge about the coin's state. This state could be prepared by flipping a fair coin, and hiding it under a hand before checking the result. The other points in this line correspond to a biased coin in a similar scenario.

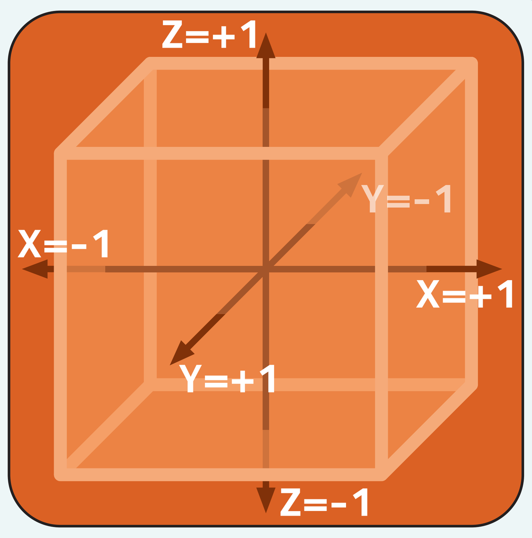


Quantum

In this framework, the set of states a single qubit can take is represented by the Bloch sphere.

Pure states, which can not be formed from mixing other states, lie on the surface. The **mixed states** are within.

It can be seen that when the one measurement outcome is totally known (such as state |0>, where <Z> = 1), the outcomes of the other measurements will be totally random as <X> = 0, and <Y> = 0.



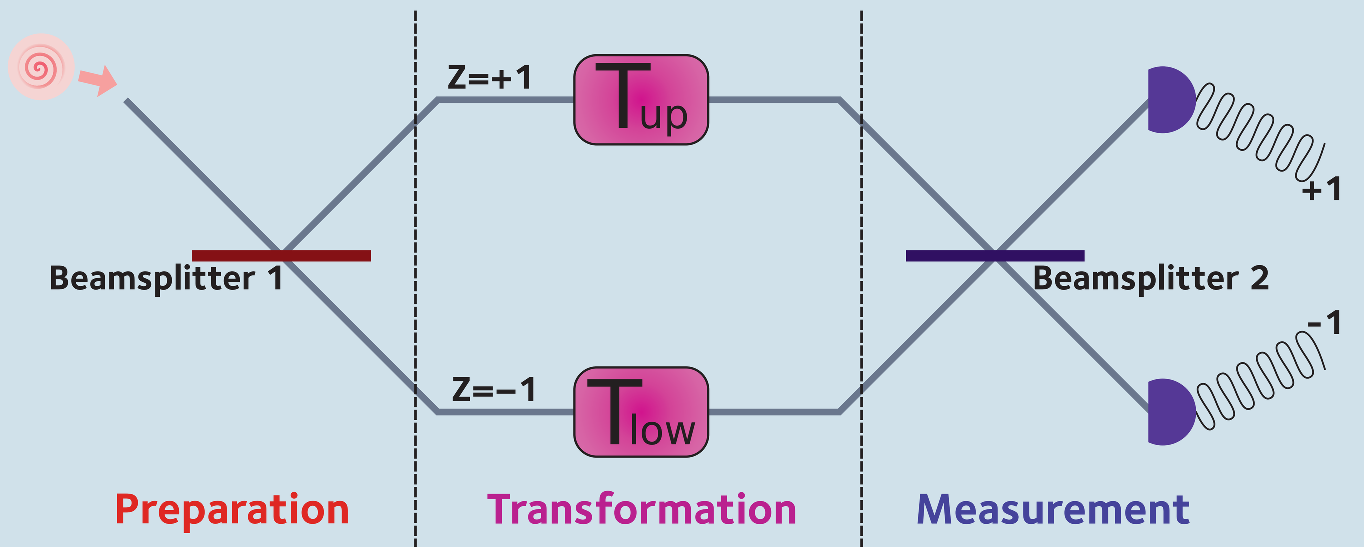
Beyond Quantum

A generalised bit (**gbit**) usually refers to the extreme case, in which every possible statistical state is allowed. In a set of binary measurements, each expectation value can take the full range of [-1,1] independently- and so the corresponding state space is a cube.

This incorporates states that are forbidden in quantum mechanics by the uncertainty principle, such as states where the outcome of the next X and Z measurement can both be predicted perfectly.

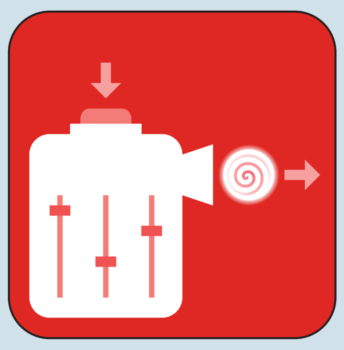
Any two states in a theory may be combined probabilistically to form a new state with proportionally combined statistics that will lie on the straight line between the original states in the state-space. As for every two states in the theory, all states on the line between them must also be in the theory, this implies all state-spaces in this framework are **convex**.

2. Generalised interferometry



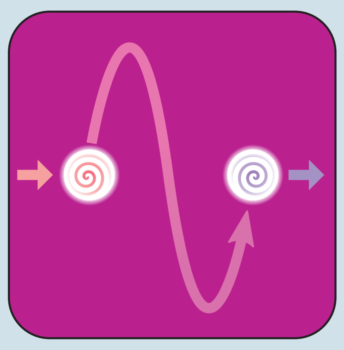
The path taken by a particle travelling through a Mach-Zehnder interferometer (above) can be described as a single bit system. One measurement (Z) states which branch the particle is in. We label the upper branch Z = +1, and the lower Z = -1. If the particle is entirely in one branch, the state corresponds to |0> and |1> in quantum theory.

In order to generalise the above experiment into to a general probabilistic theory, we consider the interferometer as having three operational stages:



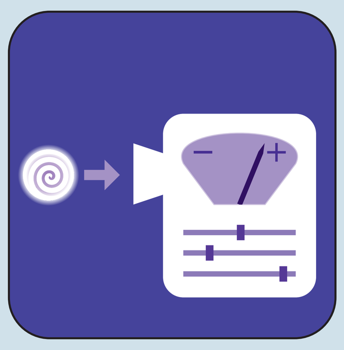
Preparation

Envision tuning a dial and pressing a button to release a particle with the desired set of statistics. In the case of the interferometer, it is useful to think of the first beamsplitter as part of the preparation process. In the quantum case, a photon would be emitted in a definite position (Z) state by the source, and then changed into a superposition (X) state by the beamsplitter. This composite process is the preparation of an X state.



Transformation

Between the creation and measurement of a state, it is possible to act on it, such that the statistics of the next measurement will be altered. This stage of the process is known as transformation. In the quantum example of the Mach-Zehnder interferometer, adding a phase delay along either of the paths would be transformation; as would completely swapping the upper and lower path, or inserting another beam-splitter.

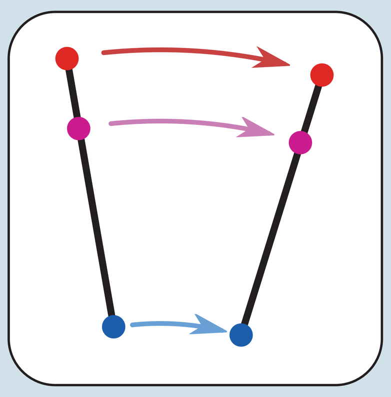


Measurement

The final stage of the experiment is to extract some classical result. As with preparation, the final beamsplitter can be considered as part of the measurement process- allowing the experimenter to choose exactly which measurement he wants to perform on the system. The outcome of the measurement is determined by the statistics which specify the state.

3. Linear Transformations

A transformation acts on every state in the entire state-space and maps it to another (or possibly the same) state. The output state-space must be contained within the input state-space. For reversible transformations, the output and input spaces are exactly the same.



All transformations within this framework are linear maps, and so may be expressed as matrices. This is because they respect **linearity of mixing**: if one is probabilistically unsure which state was input to the transformation, the same state should be found after transformation by mixing the output of each input, as would be found by applying the transformation to the composite mixed state of the inputs.

If no states are changed by a transformation, it is **trivial**. This is the same as multiplying the state as a vector by the identity matrix.

4. Branch Locality

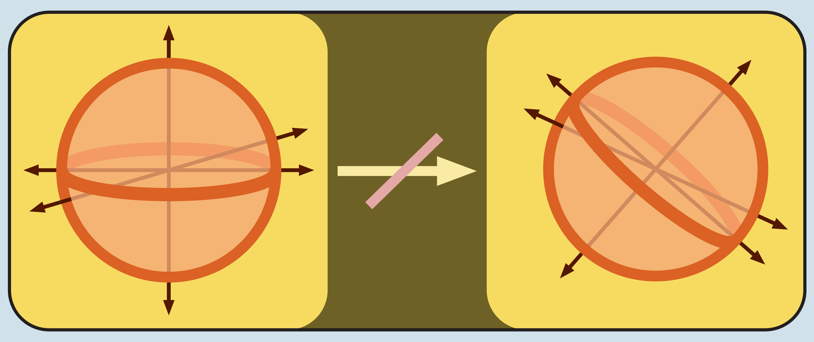
We propose a locality condition, similar in spirit to non-signalling, called **branch locality**. If a system has no probability of being in a particular location, then a transformation applied locally at that location should have no effect on the state of the system.

When the particle is in the upper branch, any transformation that is applied to the lower branch must have no effect on any of the statistics, and vice versa. In a Mach-Zehnder interferometer, the total set of allowed transformations is formed from the union of sets of local transformations on each branch, and so this constraint affects the whole set of allowed local transformations.

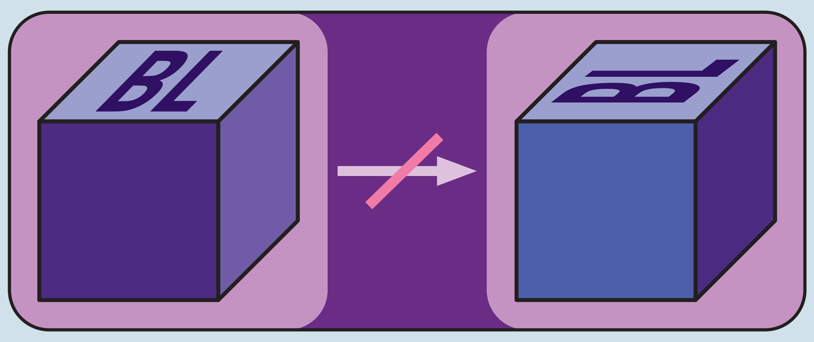
Although this statement may sound almost tautological, and should hold true in any universe with a concept of locality, we find that this implies profound consequences for the transformations allowed.

5. Consequences for interference

There are two constraints on the transformations that are possible in a spatial interferometer:



The Z statistics must not change, as the branches of the interferometer have split into disjoint regions of space, and so this would correspond to a particle jumping from one branch to another.



Branch locality implies that all states on the upper surface of the state-space Z=+1 must not be changed by an action on the lower branch (Z=-1).

Implications for gbits

Taken together, these constraints imply that for the gbit, the only transformation that may be applied at the lower branch is the identity matrix. An equivalent argument holds for the upper branch also, the total set of transformations contains only the identity element.

There are no non-trivial transformations; **local interferometry is not possible with gbits**.

Uncertainty

We know that in quantum theory, non-trivial interferometry is possible: from either branch, it should be possible to act on the state-space with SO(2)! This is possible because quantum theory has an additional constraint on the state space from the uncertainty relation, which gains back the freedom to do non-trivial transformations:

$$\langle X\rangle^2 + \langle Y\rangle^2 + \langle Z\rangle^2 \leq 1.$$

From this, we see that at the extremal planes <Z> = ±1, the state space collapses to a single point, with <X>, <Y> = 0. Even when applying a non-trivial transformation on the bottom branch, the state in the top branch remains unchanged; quantum interference works as usual.

Thus we conclude: **in order to achieve interference in a universe with a concept of locality, an uncertainty constraint is required.**

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