# Guaranteed energy-efficient bit reset in finite time

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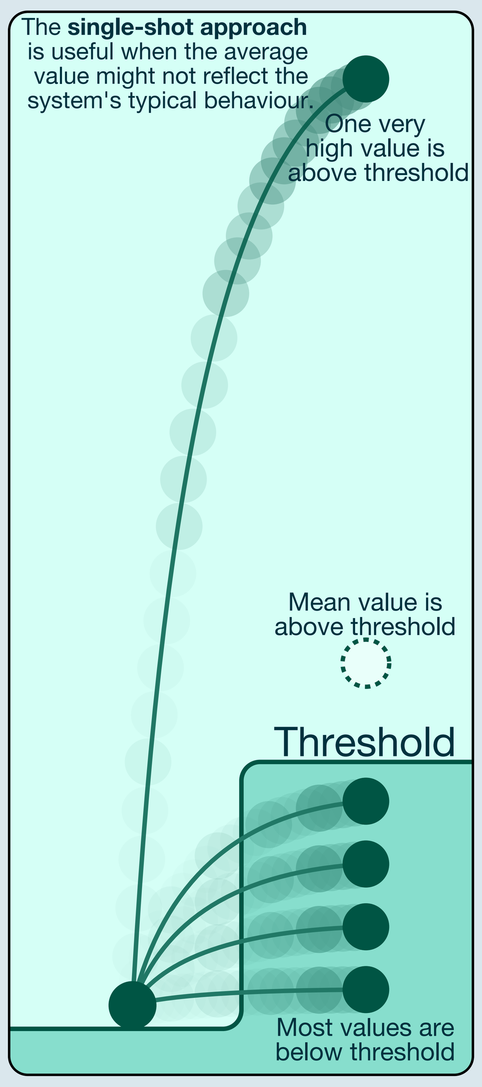
## 1. Introduction

**Landauer's principle**[1] states that resetting a bit or qubit in the presence of a heat bath at temperature $T$ costs at least $kT\ln 2$ of work, which is ultimately dissipated as heat. It represents the fundamental limit to heat generation in irreversible computers, extrapolated to be reached around 2035.

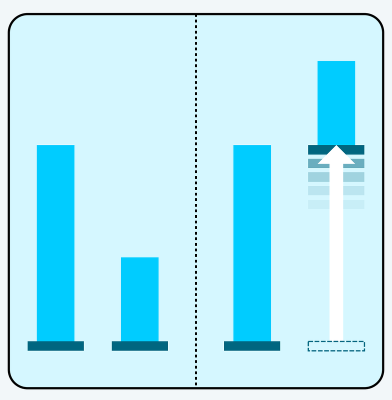
Landauer's principle is also central to the nascent **single-shot approach to statistical mechanics**, which is concerned with what is guaranteed to happen in any single run of an experiment, as opposed to the average behaviour. This is an important distinction, for example, in nano-scale computer components in which large heat dissipations in individual runs of the protocol could cause thermal damage, even if the average dissipation is moderate.




In this approach, Landauer's principle has been assumed to hold in the strict sense that one can reset a uniformly random qubit at the exact work cost of $kT\ln 2$ in each run of an experiment. This assumption is only a priori valid for quasistatic protocols which are infinitely slow, and hence unphysical. In our research[2], we investigate if this value is still appropriate for finite time processes.

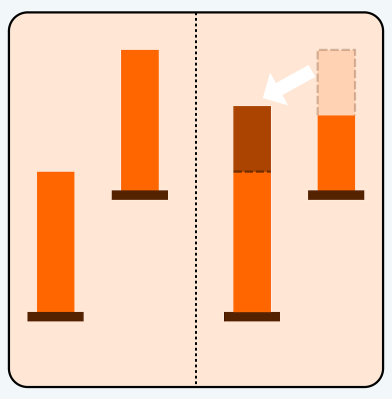
## 2. Quasistatic bit reset process

We examine a qubit system consisting of two energy levels with access to a heat bath and a work reservoir, which we will manipulate with a time-varying Hamiltonian. The evolution of the system takes place through two mechanisms, identified with heat and work[3].



### Work

Work is the energy change associated with shifting an occupied energy level, when the system is connected to a reservoir. Raising an empty level costs nothing.

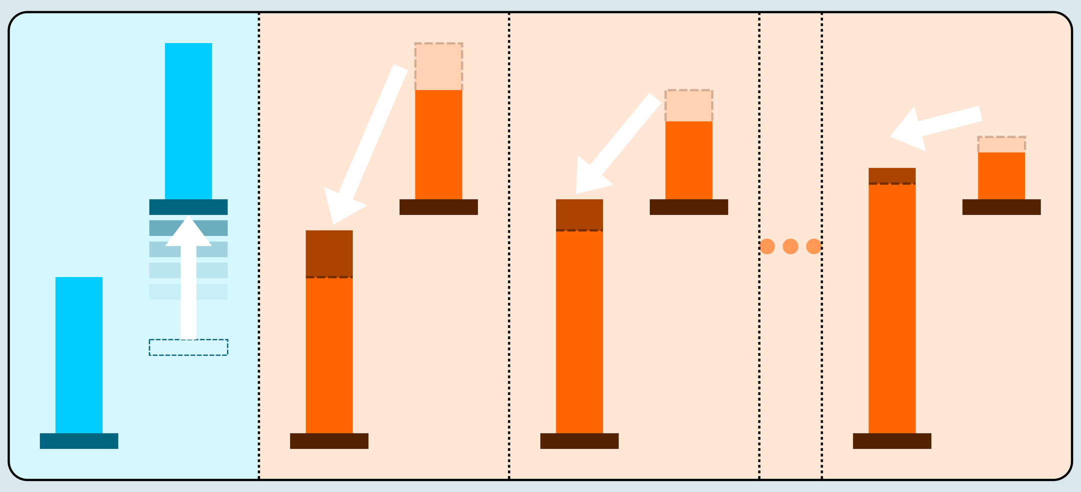


### Heat

Changing the occupation probabilities without altering the energy spectrum is associated with heat. These changes arise via thermal interaction with a heat bath.

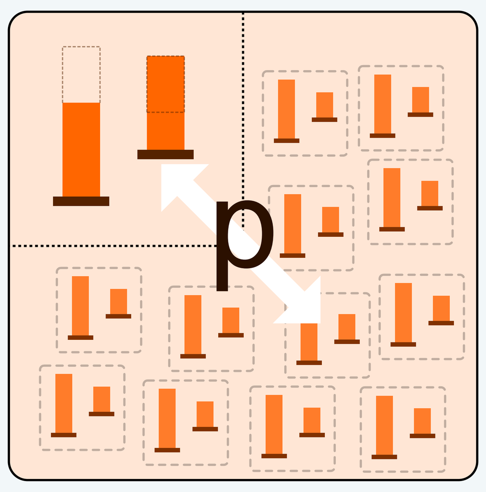
For the procedure of **bit reset** (erasure), there are initially two equally populated degenerate energy levels, equally likely to be populated. The system is coupled to a heat bath at temperature $T$, and one energy level is raised to infinity, until the lower energy level is definitely populated. In the quasi-static case this yields a work cost of $kT\ln 2$.

## 3. Extension to finite time



The process consists of a repeated cycle (above) of an instantaneous change of one energy level followed by a series of $t$ discretised thermalisation steps, each taking a fixed amount of time.

### Partial Swap




We use the partial swap model of thermalisation, equivalent to all other classical models for a two-level system[4]. The heat bath is a large ensemble of thermal states of the current system Hamiltonian, which have probability $p$ of replacing the state in a given time interval. The average effect of this is to replace the current system state with the probabilistic combination of itself and a thermal state.

### Finite time average work cost

Throughout the reset, the upper energy level is overpopulated with respect to the thermal (Gibbs) distribution. This yields an additional work penalty associated with the finite time nature of the process. The penalty falls off exponentially with the time spent thermalising, according to

$$\langle W\rangle \leq \langle W_{\mathrm{quasi}}\rangle + (1-p)^t \left(\frac{E_{\mathrm{max}}}{2} - \langle W_{\mathrm{quasi}}\rangle\right)$$

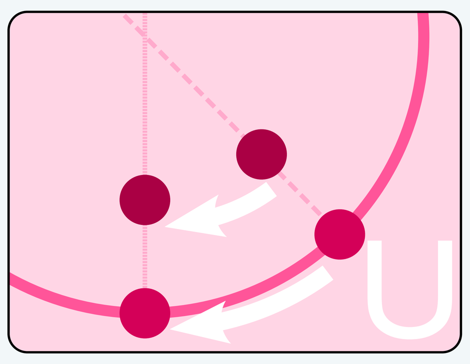
where $W_{quasi}$ is the work cost for the infinitely slow quasistatic process, and $E_{max}$ is a large but finite upper limit for the second energy level (as raising $E_{max}$ to infinity in this procedure would take an infinite amount of time).

## 4. Quantum coherences

Quantum coherences can increase the work cost of the energy level shifts by repopulating the upper energy level. We thus avoid coherence, using either of two methods:

$$E_1(t)|E_1\rangle\langle E_1| + E_2(t)|E_2\rangle\langle E_2|$$

Choosing a series of Hamiltonians that share the same energy eigenstates, only differing in energy eigenvalues, as in the standard model for Zeeman splitting.



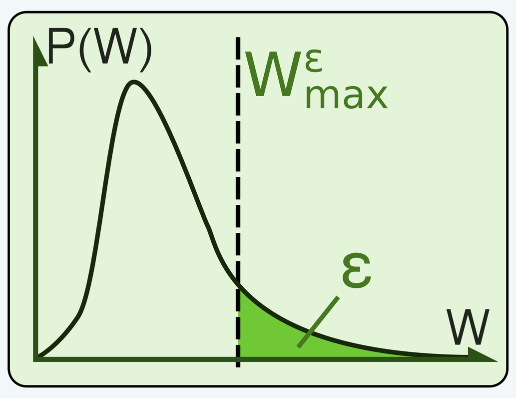

Actively undoing the coherences by applying an extra unitary on the system after the energy level shift, before the thermalisation begins.

In either case, at any point of the protocol the density matrix of the system will be diagonal in the instantaneous energy eigenbasis.

## 5. Single-shot bit reset

When fluctuations are important, it is useful to consider the probability density function for the work cost of the bit reset $P(W)$. This allows us to go beyond showing that the average work cost is close to $kT\ln 2$, and investigate what is guaranteed to be the maximum cost in every single shot of the experiment.

### Guaranteed maximum work cost




A useful parameter is the guaranteed maximum work $W^{\varepsilon}_{max}$: the upper bound on the work that could be required to perform a single reset (up to error probability ε). This value is particularly useful to know if the device would break (and the bit reset fail) when it is exceeded: either because the work must dissipate as heat that above a certain level damages the device, or if the work reservoir has a finite capacity that could be exhausted.

### Bounded deviation from mean work

We can bound the deviation ω of the work cost $W$ about its mean value. Egloff et al.[5] derive this in the quasistatic limit, but for partial thermalisation we must also take into account correlations between the system's populations at every stage:

$$P\left(|W - \langle W\rangle| \geq \omega\right) \leq 2\exp\left(-\frac{2\omega^2 P_{sw}(t)^2}{NE^2}\right)$$

where $P_{sw}(t) = 1-(1-p)^t$ is the total chance of swapping during a period of thermalisation, $N$ is the number of times we adjust the upper energy level by small amount $E$.

As $P_{sw}(t)$ is itself exponential in time $t$, the spread is **doubly exponentially supressed** in time.

### Bounded maximum work cost

We can combine our results for the average and the spread to bound the maximum work cost of erasure that will be satisfied with probability $(1-\varepsilon)$:

$$W^{\varepsilon}_{\mathrm{max}} \leq \left(1-(1-p)^t\right)\langle W_{\mathrm{quasi}}\rangle + \frac{1}{2}(1-p)^t E_{\mathrm{max}} + \frac{1}{1-(1-p)^t}\sqrt{\frac{\ln(2/\epsilon)}{2N}}\,E_{\mathrm{max}}.$$

### Conclusion

We find that one can reset a qubit in finite time at a **guaranteed** work cost exponentially close to $kT\ln 2$, not just on average but in any single shot. The optimality statements in the literature for single-shot statistical mechanics are accordingly also relevant for physical experiments, which take place in finite time.

## 6. References

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[2] C. Browne, A. J. P. Garner, O. C. O. Dahlsten, V. Vedral, arXiv:1311.7612 *(2014)*.
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[4] V. Scarani, M. Ziman, P. Stelmachovic, N. Gisin, and V. Buzek, Physical Review Letters **88**, 097905 *(2002)*.
[5] D. Egloff, O. C. O. Dahlsten, R. Renner, and V. Vedral, arXiv:1207.0434 *(2012)*.

See [2] for further references.

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