New criteria for macroscopic quantum states

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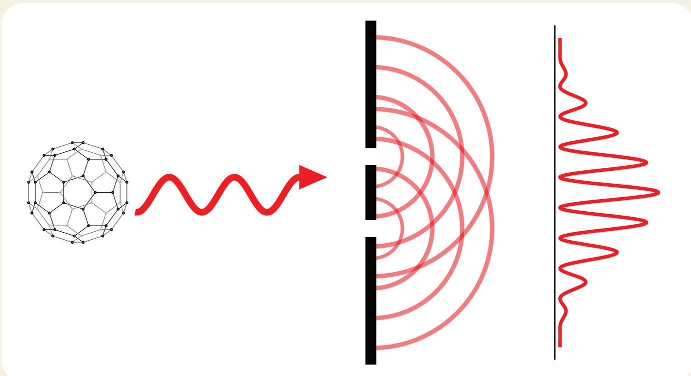
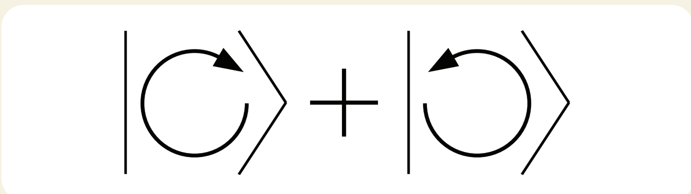
Motivation

- The search for quantum behaviour at the macroscopic scale
- States resembling Schrödinger's cat created in experiments:



Interference of large molecules (e.g. Buckminsterfullerene, C_{60}) [1]



Superposition of different currents in a superconducting circuit [2]

- We want some way of comparing the states created in different experiments - can one define an `effective quantum size'?
- In this work, we look in particular at systems of qubits and investigate the behaviour of quantum macroscopicity under local operations

GHZ states and their macroscopic quantum properties

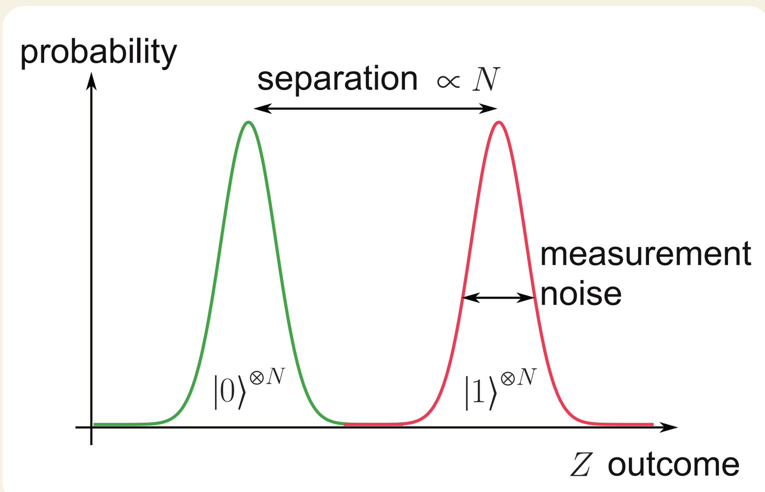
Greenberger-Horne-Zeilinger (GHZ) states with large N provide a natural set of macroscopic quantum states:

$$|\mathrm{GHZ}_N\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + |1\rangle^{\otimes N})$$

What special properties do these states have that determine their macroscopic quantum character?

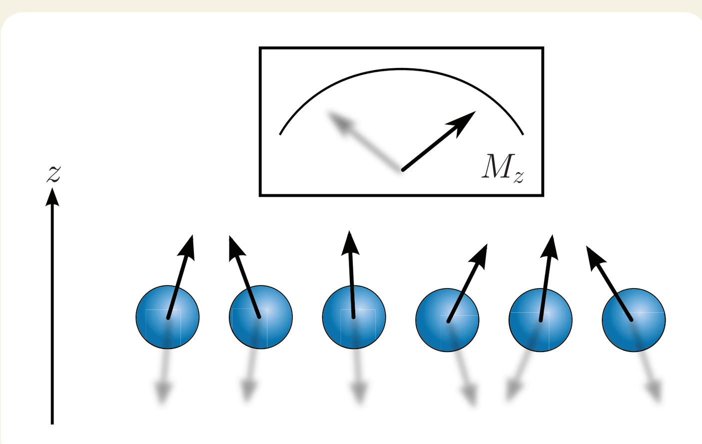
Superpositions of macroscopically distinguishable states

The states $|0\rangle^{\otimes N}$ and $|1\rangle^{\otimes N}$ are easily distinguishable by measuring $Z = \sum_{i=1}^N \sigma_i^z$ - an accessible measurement at the macroscopic scale. They remain distinguishable even with noisy measurements.



More generally, a state of the form $|\phi_0\rangle + |\phi_1\rangle$ can be considered macroscopic if $|\phi_0\rangle$ and $|\phi_1\rangle$ are macroscopically distinguishable in this sense. A very similar concept is that of local distinguishability - when one can distinguish between the branches of the superposition with high probability by measuring a single qubit. [3,4]

Fluctuations and metrology



A GHZ state displays a large variance in the Z observable:

$$\sigma(Z)^2 := \langle Z^2\rangle - \langle Z\rangle^2 = N^2$$

Classically, one would expect a variance scaling with N.

A set of spin-$\frac{1}{2}$ sites in a state like this would have highly non-classical fluctuations in its total magnetisation.

The following quantity has been proposed as a plausible effective size [5]:

$$N^*(|\psi\rangle) := \max_{A\in\mathcal{A}} \frac{\sigma(A)^2_\psi}{N}$$

Here, $\mathcal{A}$ is the set of local observables - sums of local hermitian operators (with a suitably bounded spectrum). `Local' can include bounded groups of qubits. (We would then replace N with the number of groups.) This measure can be defined for all pure states - not just superpositions of the form considered above.

One can extend the measure [5] to mixed states ρ by replacing the variance with the quantum Fisher information $\mathcal{F}(\rho, A)$. (For pure states, $\mathcal{F}(|\psi\rangle\langle\psi|, A) = 4\sigma(A)^2_\psi$.)

Characterised in this way, macroscopic quantum states are useful for metrology. If a parameter θ is encoded on a state via $\rho_0 \to \rho(\theta) = e^{-i\theta A}\rho_0 e^{i\theta A}$, then a lower limit on the error in estimating θ from the state $\rho(\theta)$ is given by the quantum Cramér-Rao bound:

$$\delta\theta \geq \frac{1}{\sqrt{\mathcal{F}(\rho_0, A)}}$$

Macroscopic quantum states, having $\mathcal{F} \propto N^2$, achieve the `Heisenberg limit' where $\delta\theta \propto 1/N$.

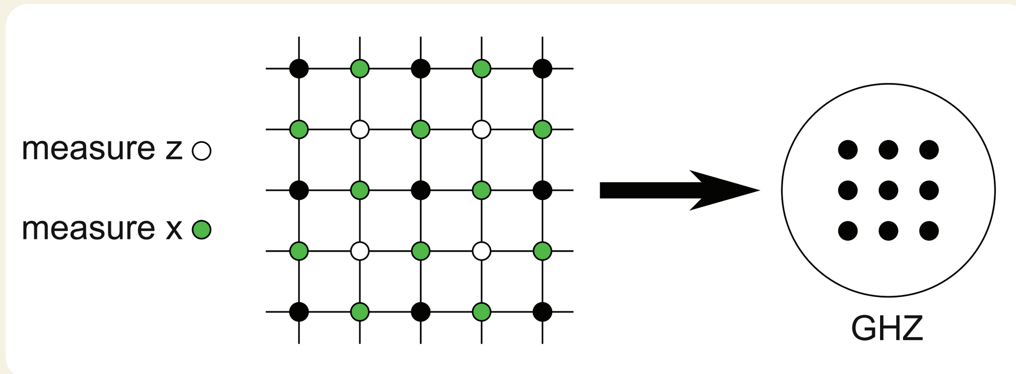
Distilling GHZ states

The authors of [6] studied `generalised GHZ states' of the form $|\psi\rangle \propto |0\rangle^{\otimes N} + |\epsilon\rangle^{\otimes N}$, where $\langle 0|\epsilon\rangle = \cos\epsilon \approx 1 - \epsilon^2/2$ for $\epsilon \ll 1$.

They found that $N^* \approx \epsilon^2 N$ according to the variance-based measure. Moreover, it is possible to probabilistically distil an exact GHZ state of average size $\langle n\rangle \approx \epsilon^2 N/2$ just by local operations. Is this a coincidence, or does it carry over to other classes of states?

Small fluctuations but large GHZ

It has been noted already [5] that one can deterministically distil a GHZ state of size $O(N)$ from a cluster state by local projective measurements.



However, the variance of any local observable can only ever scale with N.

This is because $\sigma(A)^2$ sums up all 2-point correlators - and only neighbouring regions in a cluster state are correlated.

We have found this to also be true for ground states of the Kitaev model used to define the toric code [7].

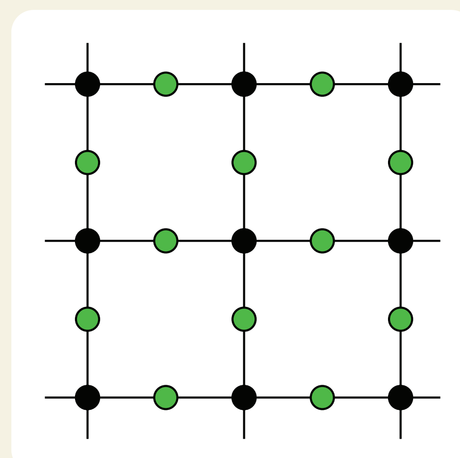
Should these classes of states be described as macroscopically quantum, despite their lack of macroscopic fluctuations?

Allowing for noisy measurements

We look at an example of GHZ distillation with imperfect (noisy) measurements. This is interesting for two reasons:

- Investigates macroscopicity under a wider set of operations
- One could object that local projective measurements require too much fine control over the system; errors may accumulate and destroy the macroscopicity

Start with the cluster state shown here. As for the state discussed above, it has $N^* = O(1)$ according to the variance measure. A GHZ state of size $O(N)$ can be projected out by the same x measurements.



Model the x measurements using the generalised measurement operators

$$M = \sqrt{1-\epsilon}|+\rangle\langle+| + \sqrt{\epsilon}|-\rangle\langle-|$$
$$\overline{M} = \sqrt{\epsilon}|+\rangle\langle+| + \sqrt{1-\epsilon}|-\rangle\langle-|$$

where ϵ is a small error parameter.

Without loss of generality, suppose the outcome is M for each measurement. Then the $n = O(N)$ `distilled' sites end up in a mixed state of the form

$$\rho = \sum_{k=1}^{2^{n-1}} p_k |\mathrm{GHZ}_k\rangle\langle\mathrm{GHZ}_k|$$

where each k corresponds to a different set of local errors.

With no errors, the final state would have $\sigma(Z)^2 = n^2$.

For the state ρ, we find that $\frac{1}{4}\mathcal{F}(\rho, Z) = n^2(1 - O(\epsilon^4))$ - so the final state has an effective size of $O(N)$. This suggests the original state might be called macroscopic.

Implications

- According to recently proposed measures, quantum macroscopicity can increase under local projective measurements - and even with imperfect measurements
- This suggests that the set of states classified as macroscopic needs to be extended
- More work is needed to understand the effects of more general types of operations, and to interpret them

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